NOAA Technical Memorandum NOS OES 005

ON THE QUASIGEOSTROPHIC OCEANIC RESPONSE TO ATMOSPHERIC PRESSURE FORCING: THE INVERTED BAROMETER PUMPING

Rockville, Maryland
April 1993

noaa National Oceanic And Atmospheric Administration

DEPARTMENT OF COMMERCE
nal Ocean Service

NOAA Technical Memorandum NOS OES 005

ON THE QUASIGEOSTROPHIC OCEANIC RESPONSE TO ATMOSPHERIC PRESSURE FORCING: THE INVERTED BAROMETER PUMPING

Chang-Kou Tai

Rockville, Maryland
April 1993

United States
Department of Commerce
Ronald Brown, Secretary

National Oceanic and
Atmospheric Administration
James Baker, Under Secretary

National Ocean Service
W. Stanley Wilson
Assistant Administrator

Office of Ocean and Earth Sciences
National Ocean Service
National Oceanic and Atmospheric Administration
U.S. Department of Commerce

The Office of Ocean and Earth Sciences provides for the understanding of the coastal and ocean environment through the conduct of applied research and development in geophysics; the measurement, analyses, and product development of ocean and lake water levels; the collection, analyses, product development, and disseminations of coastal and global marine data; and the synthesis and interpretation with numerical and mechanistic modeling of global marine data sets. The Office cooperates with the U.S. Navy in conducting oceanographic activities for defense and mixed defense-civil sector purposes and applications.

It plans, develops and coordinates NOAA participation in Federally conducted oceanographic programs and activities, and facilitates cooperative programs, projects and activities with the oceanographic research community. It monitors and analyzes oceanographic activities between NOAA and other organizations and agencies; identifies potential conflicts, overlaps, and opportunities for joint or cooperative efforts; and develops and maintains cooperative agreements, Memoranda of Understanding and other arrangements as appropriate to resolve issues and to ensure maximum benefits from programs of mutual interest. It develops and maintains inventories of oceanographic programs, projects, systems and activities of other organizations and agencies to provide a basis for integrating current and future programs, systems and activities to ensure maximum efficiency and effectiveness in meeting national goals and requirements. The Office conducts research and development; carries out theoretical studies, data analyses, and engineering development; and formulates and executes programs encompassing technological development and application to oceanography, geophysics, geodesy, and related fields.

For the Great Lakes, coastal estuaries, sea coast and oceans, the Office plans, develops, and applies numerical and mechanistic models and produces predictions, forecasts, and analysis guidance materials of oceanographic and related marine meteorological phenomena; collects, analyzes and disseminates tide and water-level observations and associated information; and computes water-level datums for hydrographic, marine boundary and other special surveys. It evaluates and improves methods of data analysis; compares and integrates existing and new classes of data and products; provides and quality controls data sets and an array of output products; and assures science and technology transfer to and from the Office's programs and projects. The Office produces and disseminates operational marine environmental forecast and analysis guidance materials; manages and supports ocean climate studies; installs and operates real-time marine data collection systems; and formulates requirements for marine data sets and for data processing and communications systems; and designs and manages computer-based systems in support of these requirements.

ABSTRACT

It is shown that the quasigeostrophic response of a homogeneous ocean to atmospheric pressure forcing consists of two parts: first, a static inverted barometer response of the sea level (which, more or less, insulates the ocean interior from direct atmospheric pressure forcing); second, a dynamical response to the inverted barometer sea level movement (called here the inverted barometer pumping in analogy to the Ekman pumping), which is one order of magnitude smaller than the Ekman pumping in mid-latitudes (two orders less in the equatorial region) and becomes even smaller for long-period forcing because the pumping is proportional to the forcing frequency. The use of the barotropic model is justified by the fact that in mid-latitudes, the response is largely barotropic for atmospheric forcing shorter than 300 days (for longer forcing periods, the inverted barometer pumping is negligible, compared with the Ekman pumping); while in the tropics, the dynamical response may be significantly baroclinic, but the pressure forcing is really insignificant. Furthermore the concept of inverted barometer pumping is valid in the stratified case as well. Two conclusions come out of this analysis. First, the dynamical response to atmospheric pressure forcing is far less important than the response to wind forcing; and the response diminishes with longer forcing periods or smaller spatial scales relative to the external Rossby radius of deformation (i.e., the rigid lid effect). Second and more importantly, irrespective of how large or small the dynamical response is, as far as one is interested in quasigeostrophic motions with the caution that high-frequency ageostrophic motions can cause aliasing problems, he is always correct in removing the static inverted barometer sea level response in order to concentrate on responses to many other sources including the dynamical response to atmospheric pressure forcing (e.g., in satellite altimetry and in pressure-treating the tide gauge data). Thus it is the desire to study the dynamic responses that compels one to remove the static response. Yet the problem is by no means solved even in the context of altimetric corrections. The ageostrophic responses to pressure as well as wind forcing, fascinating subjects in their own right, can induce aliasing problems in satellite altimetry, estimates of which are

sorely needed.

1. Introduction

It is a long-held belief that atmospheric pressure is ineffective in inducing quasigeostrophic oceanic motions. For example, in the seminal work by Veronis and Stommel (1956) on atmospheric forced oceanic motions, it is stated without proof that (compared with the wind stress curl) atmospheric pressure forcing can be neglected in the analysis of forced quasigeostrophic motions with the allowance that pressure forcing might be important in considering forced inertia-gravity waves. The natural extension of this conclusion is that the ocean interior is shielded from the atmospheric pressure by the compensating motion of the sea surface at the quasigeostrophic time and space scales, i.e., in the manner of the so-called inverted barometer wherein an increase of 1 mb in atmospheric pressure is accompanied approximately by a 1.01 cm drop in sea level. The general validity of the static inverted barometer response has been confirmed by observations. Wunsch (1972), doing a multiple regression of Bermuda sea level with local atmospheric variables, has found that between periods of about 1 day to months, the sea level indeed behaves like the inverted barometer; while for longer periods, the effects of the wind tend to dominate (see his Figs. 9 and 10). Fitting atmospheric pressure to global, monthly tide gauge data, Trupin and Wahr (1990) have found the sea level response to the pressure to be inverted barometric for periods greater than 2 months. However there are observations suggesting that the sea level response tends to deviate from the postulated inverted barometer response in the continental shelf regions (e.g., Hamon 1962, Wunsch et al. 1969, Chelton and Davis 1982).

With the advent of satellite altimetry, it is important to remove the static sea level response in order to concentrate on the dynamical response. The static response can account for more than half of the total sea level variability in mid to high latitudes (Chelton and Davis 1982; see also Ponte et al. 1991, Fig. 2 for the standard deviation of the atmospheric pressure in the North Atlantic, where it ranges from over 10 mb in high latitudes to a couple of mb in the tropics). Thus the validity of the inverted barometer response has taken on extra importance. The altimeter data can be used directly to test the validity of the inverted barometer response. Yet the results have been perplexing. Utilizing the Geosat Exact Repeat data, Van Dam (1991) has found that a linear regression yields only 60% (-0.6 cm/mb) of the theoretical response. The result holds for the global ocean as well as various smaller ocean regions. While also using the Geosat data, Wagner (1991, personal communication) has discovered that a global least-squares fit to reduce the sea level variability yields a correlation of -0.7 cm/mb. If, however, the fitting is done latitudinally, the correlation ranges from -1 cm/mb in high latitudes to no correlation in the equatorial region. The fact that these two results differ so significantly points to the difficulties in isolating the variables when they are subject to a variety of other large influences (see Tai 1990 for a brief introduction to satellite altimetry). Wunsch (1991), combining Geosat data with tide gauge data and filtering to retain only large space and time scales, has investigated the sea level response to atmospheric forcing, finding that some areas produce classical static inverted barometer response, but others suggest an amplified response.

Besides being important to physical oceanography, the inverted barometer response is important to physical geodesy in relation to the earth rotation. There, the results of comparing with observations with or without the inverted barometer correction have also been mixed, some showing improvement (Merriam 1982), while others showing degradation (Eubanks et al. 1985, Dickey and Eubanks 1986). The only logical conclusion of these findings is that the complexities of these systems (satellite altimetry and earth rotation) have precluded a simple determination. Thus analytical and numerical models have been employed to verify the inverted barometer response. Generalizing dynamic tide theory and using pressure forcing at limited number of frequencies and spherical harmonic spatial structures, Dickman (1988) has been able to conclude that the ocean is more apt to respond statically at lower frequencies or higher spherical harmonic degrees. Ponte et al. (1991) have used a barotropic numerical model for the North

Atlantic and the North Pacific basin respectively (incorporating crude bottom topography, but excluding shallow continental shelf regions or marginal seas) with forcing from observed atmospheric pressure to simulate the oceanic response. The simulation yields deviations from the inverted barometer response of only a few percent at time scales longer than 1 week, increasing to 5%-20% over the range from 1 week to 2 days for the basin-averaged estimates, but significantly larger departures from isostatic behavior are possible locally.

Lately, Ponte (1992) has given analytical solutions to atmospheric pressure forced response for a uniformly stratified ocean. He shows two seemingly surprising results. First, at most scales, the sea surface reacts as an inverted barometer to shield the interior (i.e., the interior pressure vanishes at the surface), yet the interior can respond to the pressure forcing baroclinically. Second, near the resonant scales, there are scales for which the sea surface appears not to respond to pressure forcing at all (i.e., the sea surface remains motionless). In later developments, we will show that these results can be understood perfectly in terms of the notion that pressure forcing is analogous to wind forcing and the notion of static and dynamical responses. For more realistic stratification in mid-latitudes, the response is largely barotropic for forcing periods less than 300 days with realistic spatial forcing scales (e.g., Willebrand et al. 1980).

Here we take a more basic approach. The barotropic governing equations with atmospheric pressure and wind forcing are derived for mid-latitude quasigeostrophic motions, equatorial motions, and continental shelf waves. The pressure forcing is found to pump the underlying fluid in a manner similar to the Ekman pumping due to the wind forcing. The pumping is provided by the up and down motion of the sea surface due to the inverted barometer response (called the inverted barometer pumping hereafter). It can be shown that the inverted barometer pumping operates irrespective of whether the underlying ocean is stratified or not. Thus many of our conclusions carry over to the stratified case as well. The inverted barometer pumping is one order of magnitude smaller than the Ekman pumping in mid-latitudes (two orders of magnitude smaller in the equatorial region), thus justifying the omission of pressure forcing in previous investigations on atmospheric forced quasigeostrophic ocean motions. In the continental shelf region, the inverted barometer pumping is even less important due to the emergence of a more dominant forcing by alongshore wind, whose effect is about one order of magnitude greater than the Ekman pumping. In addition, the inverted barometer pumping vanishes with lower forcing frequencies. So far as the pumping induces negligible dynamical response, the sea surface basically behaves like an inverted barometer. Even when the pressure forcing produces significant dynamical response, it is always justified to remove the static inverted barometer response in order to concentrate on the dynamical response. That is, as far as removing the static response is concerned, the relative size of the dynamic response or the wind forced response is really beside the point.

2. Theory

a. Equations of motion

A linear homogenous shallow-water β-plane model with wind and atmospheric pressure forcing is adopted here. Willebrand et al. (1980) have simulated the oceanic response to the fluctuating component of the observed wind forcing and found the dynamics to be essentially linear, thus justifying our adoption of a linear model. They also demonstrate that the oceanic response to atmospheric forcing is largely barotropic for realistic mid-latitude stratification and forcing periods less than 300 days with realistic spatial forcing scales (see their Fig. 1), whereas for longer forcing periods or the equatorial region, it will become clear that the pressure forcing is totally insignificant in comparison with the wind forcing, thus justifying the exclusion of stratification. Hence, using customary notations, the equations of motion are (ignoring bottom friction)

$$u_t - fv = -g(\eta + p_a/\rho g)_x + X/\rho H \,, \tag{1}$$

$$v_t + fu = -g\ (\eta + p_a/\rho g)_y + Y/\rho H\ ,$$

(2)

$$\eta_t + (Hu)_x + (Hv)_y = 0\ ,$$

(3)

where X and Y are the x and y component of the wind stress vector τ, p_a the atmospheric pressure, η the sea level, and H the unperturbed depth. A subscript of x, y, or t denotes the partial derivative.

Cross-differentiating (1) and (2), and substituting (3) into the resulting equation, we get the vorticity equation

$$\zeta_t - \frac{f}{H}\ (\eta_t + uH_x + vH_y)\ + \beta v = \mathbf{k}\ .\ curl\ (\tau/\rho H)\ ,$$

(4)

where ζ is the vertical component of the relative vorticity, and **k** a vertical unit vector. Note that the interior pressure is due to the combined effect of the sea level variation and atmospheric pressure. Hence, as far as the interior pressure is concerned the effective sea level, η_e, is

$$\eta_e = \eta + p_a/\rho g.$$

(5)

b. Mid-latitudes

For quasigeostrophic scales, quasigeostrophic scaling shows that (see Pedlosky 1987) to the lowest order, geostrophy holds, i.e.,

$$u = -\frac{g}{f}\,(\eta_e)_y\,, \tag{6}$$

$$v = \frac{g}{f}\,(\eta_e)_x\,, \tag{7}$$

and hence,

$$\zeta = \frac{g}{f}\,\nabla^2\,\eta_e\,, \tag{8}$$

where ∇^2 is the horizontal Laplacian. Substitution of (6)-(8) into (4) and replacement of η by η_e and p_a from (5) in (4) yield

$$\begin{aligned}(\nabla^2\eta_e - \frac{f^2}{gH}\,\eta_e)_t + \beta(\eta_e)_x - \frac{f}{H}\,J(\eta_e\,,\,H)\\ = \frac{f}{\rho g}\,k\,.\,curl\,(\tau/H) - \frac{f^2}{gH}\,(p_a/\rho g)_t\,,\end{aligned} \tag{9}$$

where J is the Jacobian. Without the forcing terms on the right hand side, (9) is the barotropic Rossby wave equation. When H is a constant, the first forcing term is simply the wind stress curl term. The second term is due to pressure forcing. With constant H, the ratio of these two terms is

$$\frac{k\,.\,curl\,\tau}{f\rho} \quad versus \quad -(p_a/\rho g)_t$$

i.e., Ekman pumping versus Inverted barometer pumping. One can also derive (9) through a different approach using the interior solution (which is quasigeostrophic) and invoking the Ekman boundary layer along with upper and lower boundary conditions.

The inverted barometer sea level response is $\eta_{IB} = -P_a/\rho g$; hence the name, inverted barometer pumping. Thus the pressure forcing is simply due to the pumping of the interior by the sea surface motion as a consequence of the static inverted barometer response. Eq.(5) can be rewritten as

$$\eta = \eta_e + \eta_{IB} \ . \tag{10}$$

If the pumping produces negligible dynamical response (i.e., η_e is approximately zero), then the sea level behaves like an inverted barometer. If, on the other hand, the inverted barometer pumping induces significant dynamical response, then the sea level is composed of two equally significant parts: the dynamical response η_e and the static response η_{IB}. Either way, it is justified to remove the static response and concentrate on the dynamic response. It can be shown that the concept of inverted barometer pumping is equally valid in the stratified case. See Section 6.9 of Pedlosky (1987, p. 365-368). Pay special attention to his Eq. (6.9.12), where the first term on the right hand side is the vortex stretching term due to the free surface, the second term is the Ekman pumping, while the third term is the inverted barometer pumping. The notion of inverted baraometer pumping is implicit in Maagard (1977).

At mid-latitudes, the mean and fluctuating part of the Ekman pumping are both of the order of 10^{-4} cm/s (see Willebrand 1978, Figs. 1 and 2; Ismer and Hasse 1987, whose North Atlantic charts are reproduced in Siedler et al. 1992, Figs. 20 and 21), while the inverted barometer pumping is of the order of 10^{-5} cm/s (i.e., 1 mb/day; see Willebrand 1978, Fig.3, and recall that we are only interested in the part of the forcing that is consistent with oceanic quasigeostrophic scales). The pressure forcing is one order of magnitude smaller than the wind forcing, thus justifying the omission of pressure forcing in most investigations. Physically this can be easily understood by noting that it is much more difficult to move the air-sea interface than the underlying interior surface. The insignificance of dynamical signals forced by pressure in comparison to wind stress has been treated in many papers (e.g., Maagard 1977, Philander 1978, Frankignoul and Muller 1979). Moreover, as the forcing frequency diminishes, the inverted barometer pumping also vanishes, in clear contrast with the Ekman pumping. This point has been made by Brown et al. (1975) and Ponte et al. (1991).

Even before any attempt to solve (9), certain bounds can be put on the dynamical response to pressure forcing. Recall that at mid and high latitudes, the static response can account for more than half of the sea level variability, while wind forcing is 10 times greater than the pressure forcing. For the sake of argument, let us just say the dynamical response accounts for 50% and one tenth of it is due to the pressure forcing. Then an upper bound of 5% can be attributed to the dynamical response to pressure forcing ; and the inverted barometer approximation is at most 10% wrong. Simple solutions of (9) are informative as to the asymptotic behavior of the dynamical response to pressure forcing. For constant H in (9) with the wind forcing ignored, a Fourier component of the pressure forcing of the form

$$p_a = P \exp[i(kx+ly-\omega t)]$$

excites sea level response of the form

$$\eta_e = A \exp[i(kx+ly-\omega t)],$$

and

$$\frac{A}{P/\rho g} = \frac{1/R^2}{k^2 + l^2 + 1/R^2 + \beta k/\omega} = \frac{1}{(k^2 + l^2)R^2 + 1 + \beta k R^2/\omega}, \tag{11}$$

where $R = (gH)^{1/2}/f$ is the external Rossby radius of deformation. Within the validity of the model, there are two interesting asymptotic limits in (11). As ω vanishes, the ratio also vanishes, which is due to the vanishing of the inverted barometer pumping. As the spatial scale becomes much less than R, the ratio also vanishes, which can be easily understood by the fact that for smaller scale motions, the sea surface plays a lesser dynamical role (which is the justification for using the rigid lid approximation), and therefore they are less easily excited by the inverted barometer pumping. R ranges from about 2000 km at 45° latitude to over 10000 km in the tropics (see Emery et al. 1984, Figs. 5a, 5b). Thus the same forcing scales become less effective as one moves equatorward. These asymptotic limits are consistent with Dickman's (1988) results, where spherical geometry rather than the β-plane is used.

The notion of pressure forcing as pumping (analogous to wind forcing) at quasigeostrophic scales explains Ponte's (1992) findings that a uniformly stratified ocean interior can respond baroclinically even though the interior is shielded from direct atmospheric pressure forcing by the compensating sea level movement (i.e., the static inverted barometer response). Following Ponte (1992), one can easily deduce from (11) at what scales the dynamical response is significant. For fixed ω and on the kl-plane, the magnitude of the dynamical response is greater or equal to the static response only inside an annulus bounded by two concentric circles with the center at $(-\beta/2\omega, 0)$ on the kl-plane, the outer radius at $\beta/2\omega$ (where $k^2+l^2+\beta k/\omega=0$) and the inner radius at $(\beta^2/4\omega^2-2/R^2)^{1/2}$ (where $k^2+l^2+\beta k/\omega=-2/R^2$). Resonance occurs on a middle circle with the radius at $(\beta^2/4\omega^2-1/R^2)^{1/2}$ (where $k^2+l^2+\beta k/\omega=-1/R^2$). For high frequencies, the annulus may become a circle instead (i.e., the inner radius is zero or imaginary) and the middle circle may not exist (note that the maximum frequency for the barotropic Rossby wave is $\beta R/2$). On the outer circle, $A/P/(\rho g)=1$ from (11); hence $\eta=\eta_e+\eta_{IB}=0$, i.e., the dynamical response matches the mirror image of the static response and the sum vanishes. Thus there is nothing strange about the seemingly odd phenomenon first noticed by Ponte (1992) that close to the resonant scales, there are scales for which the sea surface appears not to respond to pressure forcing at all. In fact there is nothing special to it at all. When one approaches the middle circle, $A/P/(\rho g)$ approaches either positive or negative infinity. From the positive side (i.e., from outside of the middle circle), the term has to go from negligibly small to positive infinity and, hence, has to pass through one, at which point the sea surface appears not to respond, but the dynamical response is there in terms of the currents. It is just that the dynamical sea level response manages to cancel the static sea level response.

With stochastic pressure forcing, resonant excitation could occur (although tempered somewhat by dissipative processes) in the real ocean. The realistic case is complicated by the basin geometry as well as bottom bathymetry, leading one to conclude that the dynamic response is geographically dependent, as confirmed by Wunsch (1991) and Ponte et al (1991).

Atmospheric pressure is more effective in generating inertia-gravity motions. Following Pedlosky (1987, p.68-69), but adding pressure and wind forcing, while ignoring the β effect, one can deduce

$$\begin{aligned}&[\nabla(gH\nabla\eta) - (\eta_{tt} + f^2\eta)]_t + fg\, J(H,\eta)\\ &= -fg\, J(H,p_a/\rho g) - \nabla[gH\nabla(p_a/\rho g)_t] + \nabla.\tau_t/\rho + fk\, .\, curl\, \tau/\rho,\end{aligned} \tag{12}$$

which for constant H becomes

$$[\nabla^2\eta - \frac{1}{gH}(f^2\eta + \eta_{tt})]_t = -\nabla^2(p_a/\rho g)_t + \frac{\nabla\, .\, \tau_t}{\rho g H} + \frac{f}{\rho g H}k\, .\, curl\, \tau, \tag{13}$$

Ignoring the wind forcing and looking for solutions of the same form as before (but for η not for η_o), one obtains

$$\frac{A}{P/\rho g} = \frac{1}{\dfrac{\omega^2 - f^2}{gH(k^2 + l^2)} - 1}\, . \tag{14}$$

There is no reason to believe the validity of the inverted barometer response at these scales. But when $\omega = f$ in (14), one gets the exact inverted barometer response. Away from this time scale, the results deviate significantly from the static response only for frequencies much higher than f or very long horizontal wavelengths (of the order of 10,000 km), or for resonance scales. Resonant excitation of basin-scale surface gravity waves (where the basin geometry and bottom bathymetry play important roles) could be more rampant than Rossby waves due to closer match of scales. These ageostropic motions forced by pressure as well as wind can cause aliasing problems even when one is primarily interested in observing the quasigeostrophic motions.

c. Equatorial region

Geostrophy should hold to within a few degree latitude of the equator, but eventually geostrophy breaks down near the equator, thus rendering (9) invalid near the equator. In its place, one can derive an equation for v alone. Following Gill (1982, p. 434-435), and adding the pressure forcing, we get for constant H

$$\nabla^2 v_t - \frac{1}{gH}\left(f^2 v + v_{tt}\right)_t + \beta v_x = \frac{k\ .\ curl\ \tau_x}{\rho H} + \frac{fX_t}{\rho g H^2} - \frac{Y_{tt}}{\rho g H^2} - \frac{f}{H}\left(p_a/\rho g\right)_{xt} + \frac{1}{H}\left(p_a/\rho g\right)_{ytt} \ . \tag{15}$$

Note that there are five forcing terms in (15) versus two in (9). The first term corresponds to the wind stress curl term, while the fourth term corresponds to the inverted barometer pumping term. The extra terms can be ignored when we are interested in low frequencies. The relative importance of these two terms is still the ratio of Ekman pumping versus the inverted barometer pumping. Yet, because of the diminishing f, the Ekman pumping is now of the order of 10^{-3} cm/s, while the atmospheric pressure variation is much diminished, amplifying the difference to at least two orders of magnitude. The mismatch of the forcing scales with the baroclinic response scales is much less in the equatorial region. Thus given strong pressure forcing, there could be significant baroclinic response. But given the total dominance of wind forcing over pressure forcing in the equatorial region, any pressure forced motions are insignificant at best, compared with wind forced motions. However the derivation is merely suggestive because the equatorial β-plane is not formally valid for the barotropic modes owing to the large external Rossby radius.

d. Continental shelf region

Geostrophy also breaks down in the continental shelf region. Assuming H = H(x), then from (4), one obtains

$$\zeta_t - \frac{f}{H}\ (\eta_t + uH_x)\ +\ \beta v = \frac{k\ .\ curl\tau}{\rho H}\ -\ \frac{H_x}{\rho H^2}\ Y\ . \tag{16}$$

Following Pedlosky's (1987, p. 630-632) scaling arguments, we get

$$\zeta = v_x = \frac{g}{f}\ (\eta_e)_{xx}\ , \tag{17}$$

$$u = -\frac{g}{f^2}\ (\eta_e)_{xt} - \frac{g}{f}\ (\eta_e)_y\ , \tag{18}$$

Substituting (17) and (18) into (16) and replacing η by η_e and p_a, it follows that

$$\begin{aligned}[\ (\eta_e)_{xx} - \frac{f^2}{gH}\ \eta_e + \frac{H_x}{H}\ (\eta_e)_x]_t + \beta\ (\eta_e)_x + \frac{fH_x}{H}\ (\eta_e)_y \\ = \frac{fk\ .\ curl\tau}{\rho g H} - \frac{fH_x}{\rho g H^2}\ Y - \frac{f^2}{gH}\ (p_a/\rho g)_t\ .\end{aligned} \tag{19}$$

The pressure forcing retains its old appearance as the inverted barometer pumping. So does the wind stress curl term. Hence the relative importance remains the same. However, there now emerges a far more dominant forcing term by the alongshore wind stress, whose ratio to the wind stress curl term is of the order of the wind spatial scale to the shelf width (i.e., 1000 km versus 100 km), hence two orders of magnitude larger than the pressure forcing.

The conclusions remain the same for the continental shelf region; namely, the response to pressure forcing consists of the static inverted barometer response and the dynamical response to the inverted barometer pumping, which is too small to be significant, compared with other forcing sources.

e. Nonlocal inverted barometer response

Ponte et al. (1991) have pointed out an overlooked aspect of the inverted barometer response, i.e., conservation of mass. For example, a hypothetical global uniform increase of 1 mb of the atmospheric pressure over the ocean would cause a uniform sea level depression of 1.01 cm according to the inverted barometer approximation. But there is no place for the water to go, thus the need for the water to rise uniformly in response to the globally averaged increase in atmospheric pressure over the ocean to cancel out the uniform depression. Hence the total response is $(\overline{p}_a - p_a)/\rho g$, where an overbar denotes a global average over the ocean (see Tai et al. 1992 for real cases of the nonlocal effect).

The nonlocal effect is a dynamical response to pressure forcing according to our classification here. If a spatially uniform pressure variation, $p_a = P(t)$, is applied to (9), the solution is $\eta_e = P(t)/\rho g$, i.e., the nonlocal effect. The interior of the ocean is not shielded from the temporal variations of the globally averaged atmospheric pressure over the ocean, rather it passes right through with no change. This may have implications for the earth rotation.

3. Summary and discussion

The quasigeostrophic oceanic response to atmospheric pressure forcing is investigated through a linear homogenous shallow-water β-plane model with justification for the assumption of linear and barotropic response, and extension to the equatorial and continental shelf region. The response is found to consist of two parts: first, a static response wherein the sea surface behaves like an inverted barometer to shield the interior from atmospheric pressure gradient imposed at the sea surface; second, a dynamical response to pumping of the interior by the up and down motion of the sea surface as a result of the static inverted barometer response, thus the name inverted barometer pumping. Note that the inverted barometer response is static only in its relation to the quasigeostrophic scales. The so-called static response is the result of dynamic adjustment through inertia-gravity motions.

The inverted barometer pumping is one order of magnitude smaller than the Ekman pumping in mid to high latitudes, and two orders of magnitude smaller in the tropics, while being two orders of magnitude smaller than the leading forcing term due to alongshore wind stress in the continental shelf region. The dynamical response tends to diminish with diminishing forcing frequencies because the pumping vanishes. The dynamical response also diminishes with smaller spatial forcing scales compared with the external Rossby radius of deformation due to the rigid lid effect. The so-called nonlocal inverted barometer response arising out of mass conservation is a dynamical response in this context. The nonlocal response allows the temporal variations of the globally averaged atmospheric pressure over the ocean to pass through the inverted barometer shield, and may have implications in the earth rotation.

The most important conclusion is that, irrespective of the magnitude of the dynamical response, it is justified and preferable to remove the static inverted barometer response if one has confidence in the atmospheric pressure field so provided for this purpose. Because the pressure forcing in our quasigeostrophic formulation is also of quasigeostrophic scales by necessity, the inverted barometer response is the response to this portion of the atmospheric pressure only. In practice, it means that one can apply the instantaneous pressure in removing the inverted barometer response if the result is subjected to subsequent low pass filtering. This is not a problem for the tide gauge, but may pose a problem to satellite altimetry, where observations at the same location may be quite infrequent. The use of analyzed pressure field produced by various meteorological agencies is a help because the analyzed field has less unwanted high frequency high wavenumber components. Attempts to model the high frequency sea level response (which should include the wind forced response as well) to avoid the aliasing problem are bound to fail precisely because of these analyzed fields' infidelity. The problem is two-fold. On the easier side, it is preferable to low pass filter the pressure field, which needs further analysis to work out the details. On the tougher side, better understating of the ageostropic responses to both the pressure and wind (which are clearly location dependent) is needed in order to estimate the size and nature of the aliasing, even though it is unlikely for one to reproduce them and remove them. The difficulties in verifying the inverted barometer response directly through the altimeter data may well be manifestations of the aliasing problem.

Acknowledgments. Discussions with L. Miller and C. Wagner have been most helpful. Comments by C. Wunsch and R. Ponte resulted in improvements. M. Davis typed the equations. This research was supported under the NOAA Climate and Global Change Program and under NASA's TOPEX Altimetry Research in Ocean Circulation Mission.

REFERENCES

Brown, W., W. Munk, F. Snodgrass, H. Mofjeld, and B. Zetler, 1975: MODE bottom experiment. J. Phys. Oceanogr., 5, 75-85.

Chelton, D. B., and R. E. Davis, 1982: Monthly mean sea-level variability along the west coast of North America. *J. Phys. Oceanogr.*, **12**, 757-784.

Dickey, J. O., and T. M. Eubanks, 1986: Atmospheric excitation of the Earth's rotation: progress da prospects. Paper presented at International Symposium on the Figure and Dynamics of the Earth, Moon, and Planets, Prague. September 15-19, 1986.

Dickman, S. R., 1988: Theoretical investigation of the oceanic inverted barometer response. *J. Geophys. Res.*, **93**, 14941-14946.

Emery, W. J., W. G. Lee, and L. Magaard, 1984: Geographic and seasonal distributions of Brunt-Vaisala frequency and Rossby radii in the North Pacific and North Atlantic. *J. Phys. Oceanogr.*, **14**, 294-317.

Eubanks, T. M., J. A. Steppe, and J. O. Dickey, 1985: The atmospheric excitation of Earth orientation changes during MERIT, in *Proceedings of the International Conference on Earth Rotation and the Terrestrial Reference Frame*, vol. 2, edited by I. Muller. PP. 469-483. Ohio State University Press, Columbus, Ohio.

Frankignoul, C., and P. Muller, 1979: Quasigeostrophic response of an infinite β-plane ocean to stochastic forcing by the atmosphere. J. Phys. Oceanogr., 9, 104-127.

Gill, A. E., 1982: *Atmosphere-Ocean Dynamics.* Academic Press, 662pp.

Hamon, B. V., 1962: The spectrum of mean sea level at Sydney, Coff's Harbor, and Lord Howe Island. *J. Geophys. Res.*, **67**, 5147-5155.

Isemer, H.-J., and L. Hasse, 1987: *The Bunker Climate Atlas of the North Atlantic Ocean*, vol. 2, *Air-Sea Interactions*, Springer-Verlag, New York.

Lighthill, M. J., 1969: Dynamical response of the Indian Ocean to onset of the southwest Monsoon. *Philos. Trans. R. Soc. London, Ser. A*, **265**, 45-92.

Maagard, L., 1997: On the generation of baroclinic Rossby waves in the ocean by meteorological forces. J. Phys. Oceanogr., 7, 359-364.

Merriam, J. B., 1982: Meteorological excitation of the annual polar motion. *Geophys. J. R. Astron. Soc.*, **70**, 41-56.

Pedlosky, J., 1987: *Geophysical Fluid Dynamics*. Springer-Verlag, 624pp.

Philander, S.G.H., 1978: Forced oceanic waves. Rev. Geophys. Space phys., **16**, 15-46.

Ponte, R. M., 1992: The sea level response of a stratified ocean to barometric pressure forcing. *J Phys. Oceanogr.*, **22**, 109-113.

Ponte, R. M., D. A. Salstein, and R. D. Rosen, 1991: Sea level response to pressure forcing in a barotropic numerical model. *J. Phys. Oceanogr., 21, 1043-1957.*

Siedler, G., N. Zangenberg, R. Onken, and A. Moliere, 1992: Seasonal changes in the tropical Atlantic circulation: observation and simulation of the Guinea Dome. *J. Geophy. Res.*, **97**, 703-715.

Tai, C.-K., 1990: Estimating the surface transport of meandering oceanic jet streams from satellite altimetry: surface transport estimates for the Gulf Stream and Kuroshio Extention. *J. Phys. Oceanogr.*, **20**, 862-879.

----, L. Miller, C. A. Wagner, and R. E. Cheney, 1992: A note on the nonlocal inverted barometer as level response. Submitted to *J. Phys. Oceanogr.*

Trupin, A., and J. Wahr, 1990: Spectroscopic analysis of global tide gauge sea level data. *Geophys. J Int.*, **100**, 441-453.

Van Dam, T., 1991: Atmospheric loading response of the solid earth and oceans. Ph. D. thesis, University of Colorado.

Veronis, G., and H. Stommel, 1956: The action of variable wind stresses on a stratified ocean. *J. Mar. Res.*, **15**, 43-75.

Willebrand, J., 1978: Temporal and spatial scales of the wind field over the North Pacific and North Atlantic. *J. Phys. Oceanogr.*, **8**, 1080-1094.

---, S. G. H. Philander, and R. C. Pacanowski, 1980: The oceanic response to large-scale atmospheric disturbances. *J. Phys. Oceanogr.*, **10**, 411-429.

Wunsch, C., 1972: Bermuda sea level in relation to tides, weather, and baroclinic fluctuations. *Rev. Geophys.*, **10**, 1-49.

---, 1991: Large-scale response of the ocean to atmospheric forcing at low frequencies. *J. Geophys. Res.*, **96**, 15083-15092.

----, D. V. Hansen, and B. D. Zetler, 1969: Fluctuations of the Florida Current inferred from sea level records. *Deep-Sea Res.*, **16**, 447-470.